

Improvements to the ATLAS – II

This list contains improvements not in Appendix 2 of “An Atlas of Brauer characters” (by Jansen, Lux, Parker & Wilson, OUP 1995), henceforth referred to as **ABC**. While some of them are new (i.e. not discovered at the time of publication of **ABC**), others were deliberately left out of **ABC** because they were not thought significant enough, or because they did not fit the format of **ABC**; but we believe that they are worth keeping on record.

As in **ABC**, we denote mathematical errors by ******* and new information by **NEW**. We use **C** to denote improvements concerning grammar or notational consistency; and **M** to refer to misprints or cases of illegibility. Changes are shown in order of appearance. New references are not shown. The inclusion or otherwise of any modification is not intended to imply anything about its inclusion in any future **ATLAS** publication.

A character table in **ATLAS** format has been obtained for ${}^2E_6(2).3$.

Introduction

- C** Page iv : Top, insert “page” before “number”.
- M** Page vii : Top left, change head of Section 1 to “Preliminaries”.
- C** : Section 2, change “to” to “from” after “differently”.
- C** Page viii : Table 1, order of $O'N$, interchange 5 and 7^3 .
- NEW** Page xi : Middle right, for sake of completeness append description of orthogonal groups in dimensions 1 and 2.
- C** Page xiii : Section 6, 2nd paragraph begins: “In the Clifford algebra, the vectors...generate a multiplicative group which is...”.
- C** Page xiv : Middle left, omit “to” in “also to specify”.
- C** Page xx : Middle right, insert “it” between “which” and “becomes”.
- ***** Page xxiii : Section 7, in middle of 4th paragraph, insert “irreducible” between “faithful” and “representation”.
- M** Page xxv : Section 5, insert i in “contans”.
- C** Page xxvi : In character table of $2A_5$, replace -0 by 0 .
- C** Page xxix : Middle right, singularize “act”.
- NEW** : In Section 18, specify notational convention where both coset and cohort are “squashed”.
- M** Page xxxi : Section 1, replace “product” by “produce”.

The Groups

- NEW** Page 4 A_6 : In “Linear” section replace in 2nd line $G.2$ by $G.2_2$.
- C** Page 12 $L_2(16)$: Singularize “Presentations”.
- C** Page 14 $U_3(3)$: In 3rd Max, replace $4.S_4:2$ by $2_+^{1+4}.S_3$.
- ***** Page 16 $L_2(25)$: In 5th Max, replace $Q_8 \times S_3$ by $(Q_8 \times 3).2$.
- ***** Page 17 : Change element orders of $4C$, $12C$, $12D$ in $4.G.2_3$ to 4, 12, 12.
- C** Page 21 $L_2(31)$: Singularize “Presentations”.
- C** Page 24 $L_3(4)$: Remove header line.
- ***** Page 28 $Sz(8)$: In “Suzuki” construction, insert before comma in 1st line “and such that when $t = 0$ we have $x = y = 0$ ”.
- C** Page 29 $L_2(32)$: Singularize “Presentations”.
- NEW** Page 30 $U_3(4)$: In 3rd Max, $N(5^2)$ is also $N(5A-D_3EF_3)$.
- ***** : Change class names $10A-B$ in $G.2$ to $10E-F$.
- C** Page 31 M_{12} : In **MINIMOG** section close bracket in 6th line.
- NEW** Page 33 : In 9th Max, replace $H.2$ by $M_8.(S_4 \times 2)$.
- M** Page 36 J_1 : Raise most of bottom line in “Graph” section.
- C** : Replace $=$ by \cong in presentation.

	C		: In 6th Max, replace D_6 by S_3 .
NEW	Page 42	J_2	: In 4th Max, replace $H.2$ by $2^{2+4} \cdot (S_3 \times S_3)$.
***	Page 52	$U_4(3)$: In 6th Max, replace $3_+^{1+4} \cdot (2S_4 \times 2)$ by $3_+^{1+4} \cdot 4S_4$.
NEW	Page 53		: 1st Max is $N(3^4) = N(3A_{10}B_{15}C_{15})$.
***	Page 55		: Change class name $12J$ in $G.2_3$ to $12K$. This also applies on page 57 and 59.
***	Page 60	$G_2(3)$: In “14-dimensional” construction, in 5th line image of e_i should be divided by 3.
	C	Page 66	$U_3(8)$: Remove the note from the map that had been suggested in Appendix 2 of ABC , since the isoclines of $3.G.3_1$ are in fact isomorphic.
	C	Page 68	$L_4(3)$: Swap the two fusion markers for $\chi_{31}-\chi_{34}$ in $2.G.2_1$.
NEW	Page 70	$L_5(2)$: In 3rd and 4th Maxes, $N(2^6)$ is also $N(2A_{21}B_{42})$.
	C	Page 73	$U_5(2)$: In 4th Max, remove brackets round AB and CD .
NEW	Page 74	$L_3(8)$: In 3rd Max, $N(7^2)$ is also $N(7A-F_3GH_2IJK_3)$.
***	Page 77	$Sz(32)$: In “Suzuki” construction, insert before comma in 2nd line “and such that when $t = 0$ we have $x = y = 0$ ”.
NEW	Page 79	$U_3(9)$: In 4th Max, $N(5^2)$ is also $N(5A-D_3EF_3)$.
	C	Page 80	HS : 2nd, 3rd, 6th, 7th, 9th and 11th Maxes respectively fix $233-2323$, $233-2332$, $233-2433$, $233-2433$, $233-2343$ and $233-2433$ points. For the 6th, 7th and 11th this is a further change, in all cases designed to conform with a convention that in $abc-defg$ the sums of (the vector corresponding to) d with a, b, c are e, f, g respectively.
	C	Page 81	: Replace $*7$ by $*2$ in χ_{42} .
***	Page 82	J_3	: In “Unitary” construction, the definition of C_Z only applies when $z \neq 0$: the image of $e(0)$ is $e(0)$.
	C	Page 83	: Replace $r6 *$ by $r6 -r6$ in χ_9 .
	C	Page 89	$O_8^-(2)$: Pluralize “Presentation”.
***		${}^3D_4(2)$: In “Jordan” construction, replace top row (but not 1st column) occurrences of $\frac{1}{4}s$ by $\frac{1}{4}\bar{s}$ in 768 matrix orbit.
	M		: In line after display of roots in same section, delete comma before slash.
NEW			: New presentation for G is linear Coxeter graph $\langle a6b3c3d \mid ((ab)^2c)^6, (a^{bc}(a^{b^c b^a}abcd)^2)^3, (abcd)^{21} \rangle$. Omitting the last relator gives a presentation for $S_4 \times G$.
NEW			: In 7th Max, $N(7^2)$ is also $N(7A-C_4D_4)$.
***	Page 97	$G_2(4)$: In 2nd Max, delete entry in last column.
	C	Page 100	McL : 3rd, 7th and 10th Maxes respectively fix $223-2324$, $223-2443$ and $223-2434$ points. For the 7th this is a further change. See amendment to Page 80 above.
NEW	Page 104	He	: 5th Max is split.
	C	Page 109	$O_7(3)$: Pluralize “Presentation”.
***			: In 14th Max, replace $2E$ by $2F$.
	C	Page 113	$S_6(3)$: Move “Specifications” to right.
NEW			: 2nd Max is $N(3^6) = N(3AB_{13}C_{39}D_{78}E_{234})$.
NEW			: 3rd Max is $N(3^3) = N(3AB_4C_3D_6)$.
***			: In 7th Max, extensions are split.
NEW	Page 115	$U_6(2)$: $N(2^9)$ is also $N(2A_{21}B_{210}C_{280})$.
	C		: 5th, 6th, 11th and 12th Maxes respectively fix $222-2323$, $222-2332$, $222-2433$ and $222-2343$ points. See amendment to Page 80 above.
***	Page 123	$R(27)$: In 1st Max, replace both occurrences of 3^{3+6} by 3^{3+3+3} .
	C		: Insert separator between $R(27)$ and $S_8(2)$.
	C	$S_8(2)$: Pluralize “a presentation”.
NEW			: 7th Max is $N(2^6) = N(2A_7B_7C_{21}E_{28})$.
	C	Page 129	Suz : Replace $r10 *$ by $r10 -r10$ in χ_{40} .
	C	Page 131	: Pluralize “Presentation”.

- *** : 12th Max is non-split.
- C Page 140 $O_8^+(3)$: Pluralize “a presentation”.
- NEW : Maxes 7-9 are also $N(3^6)$.
- *** : In 8th and 9th Maxes, replace $L_4(3) \times 2$ by $L_4(3):2_1$.
- NEW Page 146 $O_{10}^+(2)$: In 2nd Max, $N(2^8)$ is also $N(2A_{135}B_{120})$.
- NEW : In 3rd and 4th Maxes, $N(2^{10})$ is also $N(2A_{155}C_{868})$.
- NEW Page 147 $O_{10}^-(2)$: In 1st Max, $N(2^8)$ is also $N(2A_{119}B_{136})$.
- NEW : In 5th Max, $N(2^6)$ is also $N(2A_{35}C_{28})$.
- NEW : In 6th Max, replace $N(2^3)$ by $N(2A^3)$.
- C : In last Max, $N(3A_2D_2)$ is also $N(3^2)$.
- NEW Page 154 Co_2 : In 2nd Max, $N(2^{10})$ is also $N(2A_{77}B_{330}C_{616})$.
- *** Page 160 Fi_{22} : In the additional five columns, on p. 310 of **ABC**, the centralizer orders of $12L$, $12M$ and $12N$ are 3456, 1296 and 576 respectively.
- *** Page 163 : 7th Max is $(2 \times 2_+^{1+8}):U_4(2) : 2$ extending to $H:2$.
- NEW : In 8th Max, replace $N(2^4)$ by $N(2B^4)$.
- C Page 172 $F_4(2)$: Replace -0 by 0 in χ_{96} on $10A$.
- NEW Page 177 Fi_{23} : In 6th Max, $N(2^{11})$ is also $N(2A_{23}B_{253}C_{1771})$.
- NEW : In 11th Max, $N(2^6)$ is also $N(2A_7B_{21}C_{35})$.
- C Page 182 Co_1 : Near bottom, pluralize “portion”, “group” and “parenthesis”.
- *** Page 183 : Maximal subgroup $5^3 : (4 \times A_5).2$ is in fact $5^3 : (4 \times S_5)$.
- *** Page 191 ${}^2E_6(2)$: 16th Max is $3^{1+6}:2^{3+6}:3^2:2$.
- C : In last Max, replace 3^2Q_8 by $3^2:Q_8$.
- NEW Page 217 B : New Maxes M_{11} , $L_3(3)$, $L_2(17) : 2$ and $L_2(11) : 2$. List is now complete. Furthermore, in the group $(S_6 \times L_3(4) : 2) : 2$ quoted in the original list of amendments, the 1st “:2” is in fact “:2₂”.
- C Page 230 M : The symbol \mathbb{M} is now used for the Monster.
- C : Pluralize “Presentation”.
- NEW : New presentation for G has generators s, t, v, x and relators $s^6, t^3, (st)^4, (s^2t)^4, (s^3t)^3, [s^2, (ts^2t)^2], [v, ut^{-1}], [v, u^3su^{-2}], v^2, [v, v^t], (vu)^{13}, [vx, ut^{-1}], [vx, s^{u^3}], x^3, (v^u vx)^2, (x^{-1}x^s)^2, (xt^{-1})^{12}, (u^{-6}xu^6s)^6(sux^{-1}u^{-1})^6s^{-1}$ and $((xv^{u^4}v^{u^{10}})^3u)^{13}$. s and t are the generators for $L_3(3)$ used on page 13, and their permutation action on the points of the projective plane can be written as $s = (1, 2, 5, 9, 8, 7)(3, 12, 4)(10, 11)$ and $t = (0, 12, 3)(1, 2, 4)(5, 7, 11)(6, 9, 8)$. v is the product of all points of the projective plane except the one corresponding to 0, and x is the product of that point with the line corresponding to $(0, 1, 3, 9)$. Omitting the penultimate or last relators, or both, gives the groups $G \times L_3(3)$, $G \times G$, and $G \times G \times L_3(3)$ respectively.
- M : Replace , by . in last line before list of groups.
- NEW Page 231 : A list of groups which may be involved or contained in the Monster has been compiled. See below.
- NEW Page 234 : The list of Maxes that are 2-local or normalizers of products of two or more simple groups is complete. New Maxes $L_2(19):2$, $L_2(29):2$, $L_2(41)$, $L_2(59)$ and $L_2(71)$ – the last two implying that 59.29 and 71.35 are not maximal.

Additional Information and Bibliography

- C Page 239 : Pluralize “Order”.
- NEW : Insert sentence: groups whose full character tables are in the **ATLAS** are starred, while a bracketed star is appended to other **ATLAS** groups and to those for which all or part of the character table exists in **ATLAS** format. Also, insert relevant stars, with or without brackets, in pages 239-242.
- *** Page 241 : Outer automorphism group of $G_2(9)$ is 4.
- M Page 247 : line 3: Replace 243 by 323.

Which simple groups are contained or involved in the Monster?

We divide all simple groups whose order divides that of \mathbb{M} into the following four types:

(a) The following groups are subgroups of \mathbb{M} and their occurrences are completely classified: C_p (p prime dividing $|\mathbb{M}|$), A_n ($5 \leq n \leq 12$), $L_2(q)$ ($q = 7, 8, 11, 17, 19, 23, 25, 29, 31, 41, 49, 59, 71$), $L_4(3)$, $U_3(5)$, $U_5(2)$, $S_4(4)$, $S_n(2)$ ($n = 6, 8$), $O_7(3)$, $O_8^+(q)$ ($q = 2, 3$), $O_n^-(2)$ ($n = 8, 10$), $G_2(3)$, ${}^3D_4(2)$, ${}^2F_4(2)'$, M_{12} , Fi_{23} , He , HN .

(b) The following groups are subgroups of \mathbb{M} but their occurrences have not been completely classified: $L_2(q)$ ($q = 13, 16$).

(c) The following groups are known to be involved in \mathbb{M} but not contained: $L_2(81)$, $L_3(q)$ ($q = 4, 5$), $L_5(2)$, $U_4(3)$, $U_6(2)$, $O_8^-(3)$, $O_{10}^+(2)$, $G_2(4)$, $F_4(2)$, ${}^2E_6(2)$, M_n ($n = 22, 23, 24$), Co_n ($n = 1, 2, 3$), Fi_{22} , Fi'_{24} , J_2 , Suz , HS , McL , B .

(d) The following groups are not involved in \mathbb{M} : A_n ($13 \leq n \leq 32$), $L_2(q)$ ($q = 27, 32, 47, 64, 125, 169, 1024$), $L_3(q)$ ($q = 7, 9, 16, 25$), $L_4(q)$ ($q = 4, 5, 7, 9$), $L_5(q)$ ($q = 3, 4$), $L_6(q)$ ($q = 2, 3, 4$), $U_4(q)$ ($q = 4, 5, 8$), $U_n(4)$ ($n = 5, 6$), $S_4(q)$ ($q = 5, 7, 8, 9$), $S_6(q)$ ($q = 3, 4, 5$), $S_8(3)$, $S_n(2)$ ($n = 10, 12$), $O_7(5)$, $O_9(3)$, $O_{10}^+(3)$, $O_{12}^\pm(2)$, $Sz(8)$, $Sz(32)$, $G_2(5)$, J_n ($n = 1, 3$), $O'N$, Ru .

The number of changes of each type is as follows: 22*** 32NEW 39C 7M.

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